BIOS 6244 Analysis of Categorical Data
Assignment 1 Solutions

(1) (a) Work Exercise 1.8, p. 14 in our textbook using the "usual" approximate methods. (See attached.)

Solution

Let π = Pr(greater relief with new analgesic).

a. We wish to test H_0: $\pi = .5$ vs H_a: $\pi \neq .5$. Using the normal approximation to the binomial, the test statistic is

$$z = \frac{p - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{N}}} = \frac{.6 - .5}{\sqrt{\frac{.5(1-.5)}{100}}} = 2.00$$

Thus, p = 2Pr(Z ≥ 2.00) = 2(.023) = .046 by SimCalc. Since p < .05, reject H_0 and conclude that a significantly greater proportion of women find relief with the new drug than with the standard one.

b. We want to find an approximate 95% CI(π).

Again using the normal approximation to the binomial, the formula to use is

$$p \pm 1.96\sqrt{\frac{p(1\text{-}p)}{N}} = .6 \pm 1.96\sqrt{\frac{.6(.4)}{100}} = .6 \pm .096 = (.504, .696).$$

Since .5 $\notin$ CI(π), we conclude that the new drug is better than the standard.

(b) Find the exact p-value and an exact 95% CI (π).

Solution

To find the exact p-value, calculate

2Pr(Y ≥ 60 | n = 100, π = .5) = 2(.0284) = .057 (rounded to 3 decimal places) by SimCalc. Since .057 > .05, do not reject H_0 and conclude that the new drug is no better than the standard.

From SimCalc, an exact 95% CI(π) is given by (.497, .697). Since .5 $\in$ CI(π) [just barely], conclude that the new drug is no better than the standard.

Note that the conclusions for the exact method and the "usual" approximate method do not agree. The correct conclusion is the one based on the exact

method, of course. This illustrates the importance of using exact methods whenever they are available.

(2) Suppose that in a small pilot study with only 10 subjects, 4 women with dysmenorrhea reported greater relief with the standard and 6 preferred the new analgesic.

(a) Find the exact p-value.

Solution

The exact p-value is given by

$2Pr(Y \geq 6 \mid n = 10, \pi = .5) = 2(.377) = .754$ (rounded to 3 decimal places) by SimCalc. Since $.754 > .05$, do not reject H_0 and conclude that the new drug is no better than the standard.

(b) Find an exact 95% CI(π).

Solution

From SimCalc, an exact 95% CI(π) is given by (.262, .878). Since $.5 \in$ CI(π), conclude that the new drug is no better than the standard.

(c) Compare the CI's you obtained in 1(b) and 2(b) above. Comment.

Solution

The interval in 2(b) is much wider than the interval in 1(b) [width of .616 vs. width of .200]. This is to be expected because of the difference in sample sizes (n = 10 vs. n = 100) and illustrates the importance of having an adequate sample size. (Note, however, that neither exact CI indicates that the new drug is any better than the standard.)

(3) Work Exercise 1.9, p. 14 in our textbook. (See attached.) Follow the hint provided by Agresti, using the "usual" method for finding an approximate 95% CI (π).

Solution

(continued on next page)

Start with the formula for standard error based on knowing the true proportion π:

$$SE(\hat{\pi}) = \sqrt{\frac{\pi(1-\pi)}{N}}.$$

The "±" term in the approximate 95% CI(π) is 1.96SE($\hat{\pi}$), so following Agresti's hint, we have

$$1.96\sqrt{\frac{.75(1-.75)}{N}} = .08 \;\Rightarrow\; \frac{(1.96)^2(.75)(.25)}{N} = (.08)^2 \;\Rightarrow\; N = \frac{(1.96)^2(.75)(.25)}{(.08)^2}$$

$\Rightarrow N$ = 112.5 $\rightarrow$ 113, rounded up to nearest integer.